2.4. Lifetime assessment for capacitors in EPS application

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ABSTRACT

Usually the electrical engineer is obligated to prepare the lifetime calculation for capacitors and review them with automotive OEM. Some rules of using life equation especially for variable loads and temperature are not well popularized and equation itself ask for understanding. Lifetime expectancy a historical note with discussion about lifetime equation and its roots of origin, 10 degree rule and connection with reaction speed. Open question to industry pointing out missing data and unstandardized specification. The article will present search for lifetime equation background, showing different approaches and trying to simplify its understanding. Since some information are hard to judge without expertise knowledge, thus the open questions to industry will be addressed.

"If I have seen further than others, it is by standing upon the shoulders of giants" – Sir Isaac Newton.

https://en.wikipedia.org/wiki/Standing_on_the_shoulders_of_giants

LIFETIME CALCULATION OF CAPACITORS

Usually the electrical engineer is obligated to prepare the lifetime calculation for capacitors and review them with automotive OEM. It's because failure of capacitor for example bulk one at battery site will bring highest severity and lack of control. On the other hand the degradation of capacitance or increase of ESR can bring instability of EPS inverter and result again in loss of assist.

Some rules of using life equation especially for variable loads and temperature are not well popularized and equation itself ask for understanding. Seems philosophical yet is important trying to handle the equation background.

Imagine situation, You got a topic or a subject which You do not know more about, and You are searching for understand. It's not easy, the way has many obstacles, You don't have any to ask, nor support.

That's a case of lifetime equation, when You search on Your own.

Of course when You dig more, more question starts to grow, and tree of questions do not convolute… yeah…

We searched lot of papers and books, that's a story about… From this position it reminds standing on shoulders for giants the work of researchers during XIX/XX century period.

Concludes presented here are merely guessing not a scientific proof.

History of life analysis is old. One need to go back about 100 years before to get some answers.

Before searching Arrhenius law, one should mention accelerated test is very old discipline, with one of the pioneers like August Wohler and Oli Basquin. Its because their idea of using exponential plot, reminds something common with Arrhenius plot used today. Idea of linearization stress-strength graph by transformation.

The discipline came from train (locomotive) axels which were broken during load-vibrations, engineers analyzed how they survive (deterioration, fatigue).

Today lifetime is linked with reliability (started during 50ties and WWII wartime) with its roots pointing at Benjamin Epstein, John H. K. Kao and many others. However life expectancy was known far longer before, In 30ties Edwin Kurtz and Robley Winfrey published survivor curves and analyzed life data. (https://doi.org/10.31274/rtd-180813-15936)

E. B. Kurtz, *"Life Expectancy of Physical Property",* Ronald Press, New York, 1930;

E. B. Kurtz and R. Winfrey, *"Life characteristics of physical property"*

However all started at other discipline the statistics, and we can say insurances. Benjamin Gompertz obtained life time plots, which are very similar to bathtub curve. And bathtub curve remind life time during start, normal and wear out period.

The idea of Gompertz is something striking, the geometrical series which he used, keeps in nature. Nature do not use gaussian addition, but rather prefers multiplying. (https://people.math.ethz.ch/~stahel/talks/lognormal.pdf) That was idea behind von Kapteyn study in 1903 about log-normal distribution, which was not popularized until Bell Labs went back with reusing Kapteyn's idea. (https://zbmath.org/34.0268.03)

On the other hand, MLCC today are grained ceramics, in the 80-ties Japanese found he relation grain size, mechanical vs electrical strength correlate both and are kind of Weibull. (https://doi.org/10.1111/j.1151-2916.1984.tb09617.x)

It was also nothing new, since Rosin & Rammler obtained grain size statistics which was a predecessor for Weibull distribution. And statisticians know about it. (https://doi.org/10.1201/9781420087444))

In 40ties a book about sand-prism says: nature like multiplication (ratios), although Egyptians used linear ruler, but nature follows logarithmic scale. So several studies started no in electricity but in mineralogy and geology. It could be not obvious from first look, but all materials get from mother earth. Mining is an old business. (https://doi.org/10.1007/978-94-009-5682-7)

As one can see the lifetime study is nothing new, it started with Gompertz Benjamin saying that nature like multiplication (similar like Kapteyn treat the lognormal). And his data of peel death are very close matched to the bathtub curve.

In 30ties Edwin Kurtz edited his book about life expectancy, and Robley Winfrey analyzed many survivor curves (about 176 curves of 18 types). Luckily in 1955 Kurt Stange developed the "Abgangslinie" and popularized Weibull method to interpolate the life results. It was a good point to obtain one common application the Weibull instead of hundred curves! It was and is up today a biggest strength of Weibull. (K. Stange Mitt. F. Math. Statistik u. Ihre Anw. 7 1955)

We can now switch back to Arrhenius because, lifetime equation is related to Arrhenius. For first sight it looks strange, where such an equation came from? Why it's called Arrhenius?

Taking a look at LT equation and comparing with Arrhenius doesn't look obviously the same.

$$L = Lo \times 2^{\frac{125 - Ta}{10}} \times 2^{\frac{7 - \Delta T}{10}} \quad ? \quad k = B \cdot T^n \cdot e^{-\frac{E_A}{R \cdot T}}$$

L : estimation lifetime (h) Ta : ambient temperature (℃)
Lo : guaranteed lifetime (h) Tc : case top temperature (℃)
⊿T : self-heating by ripple current (℃) Tc-Ta

Let's try unhide it. Usually Electrical engineer doesn't know Arrhenius, since it came from chemistry… But what has reaction speed in common with aging?

Its good point out that industry successfully linked reaction speed with aging behavior.

As we found that 10deg rule in chemistry is called RGT rule in chemistry. Finding RGT was a big step to search for some relations and link the missing dots… !!!

$$\frac{k_{t_1}}{k_{t_2}} = n^{\frac{t_1 - t_2}{10}}$$

(Bodenstein, M. (1899) https://doi.org/10.1515/zpch-1899-2920)

10deg rule in chemistry but why in lifetime…?

10deg rule came from chemistry famous RGT rule into insulation aging. Many old books especially German shows such a temperature relation.

It was end of XIX century when Leyden chemist Jacobus Van't Hoff with several assumption concludes that reaction doubles or triples in every 10deg rise of temperature. The name RGT (from German RGT means "Reaktion Geschwindikgeit" which is Reaction Speed) is mention by C.P. Stuart as early as in 1912. (https://de.wikipedia.org/wiki/RGT-Regel) (Cohen Stuart, C.P., KNAW, Proceedings, 14 II, 1912)

Worth to notice that before van't Hoff formulated the 10deg law circa 1896 the another Belgian famous chemists Walthère Victor Spring showed (1887) a result which reminds todays lifetime formula with a data from polish researcher Boguski. The reaction speed analysis of Józef Jerzy Boguski influenced than van't Hoff (!)

$$v = k \cdot 2^{\frac{t}{20}}$$

(https://orbi.uliege.be/bitstream/2268/68035/1/Spring_%C3%9Cber%20den%20Einfluss%20der%20Temperatur.pdf)

One guess that 10C base in lifetime equation also behaves like RGT. Life halves when temp increase 10C and doubles when temp decreases 10degrees. But... is it true?

On the other hand 10deg was mention in insulation resistance by Graham Lee Moses, and in paper by thermal runaway Roger Stout in example of Schottky diode leakage growing with temperature like RGT.

(https://www.eetimes.com/predicting-thermal-runaway-part-2/,

https://books.google.pl/books/about/Insulation_of_Electrical_Apparatus.html?id=uqNRAAAAMAAJ&redir_esc=y)

10 deg rule seems something common, yet it's not popularized to google it out easily ;-)

$$L = Lo \times 2^{\frac{125 - Ta}{10}}$$

(Generic Life equation)

$R_2 = R_1 \times (0.44)^n$ for class A insulation.
$R_2 = R_1 \times (0.63)^n$ for class B insulation.

where R_1 = resistance at temperature 1.
R_2 = resistance at higher temperature 2.
n = temperature difference (2 − 1) divided by 10.

(Graham Lee Moses)

$$I = I_0 2^{\frac{T}{10}} \qquad \text{(eq. 4)}$$

(Roger Stout)

Track of Arrhenius roots

One of the first Arrhenius plot for capacitor aging were found in Berberich 1948 paper. (https://doi.org/10.1021/ie50457a033) There is no answer why he used Arrhenius plot. Regarding statistics the gaussian peak of life in 1942 Scott paper (bell shape) this paper doesn't say its normal distribution yet it is after they mention rot mean square. (10.1109/T-AIEE.1942.5058549) We see the temperature influence aging (accelerates), while statistics is important tool to analyze aging distribution.

The search is nonlinear process.

Some chemical researches showing the AH in collision theory and thermionic currents… important today, but not for lifetime at first insight

The story of chemical rate of reaction is very old starting from Pfaundler, Berthelot, Jacobus van't Hoff, Svante Arrhenius, Max Bodenstein , Max Trautz collision theory, Rene Marcelin statistics. The review made by Keith Laidler shows the AH factor was settled about 1910 in popular usage while about 1920 statistician calculated the AH in relation to a kind of Boltzmann statistics obtaining the exp -1/T relation. (https://doi.org/10.1021/ed061p494)

Chemists analyzed reaction speed long time ago, there are many equations -> many temperature coefficients. For example Arrhenius (AH) one, Berthelot one, Harcourt Essom and many others.

Germany popularized Van‘t Hoff not Arrhenius at start, since it was van‘t Hoff (from Leyden) whom concluded reaction speed basis on many other results (including Arrhenius one). Sometime the 1/T factor is called Boltzmann factor. Again it was because the Arrhenius studied under Ludwig Boltzmann, and also worked with Jacobus van't Hoff. In other words the Maxwell-Boltzmann distribution was linked with reaction theories.

Boltzmann im Kreis von Kollegen in Graz 1887. Stehend von links: Walther Nernst, Heinrich Streintz, Svante Arrhenius, Richard Hiecke; sitzend von links: Eduard Aulinger, Albert von Ettingshausen, Ludwig Boltzmann, Ignaz Klemenčič, Viktor Hausmanninger (source: https://de.wikipedia.org/wiki/Ludwig_Boltzmann)

AH in many other fields current thermionic, insulation resistance…a common thing… but still missing how it came into life equation?

When one tries to find the AH relation it finds it was well known in thermionic emission and insulation resistance. Striking is that thermionic current has very same equation as Arrhenius once. That's strange for a start, but it helps understand how such an idea went into insulation aging theories.

Isolation resistance and 10deg rule demystifying

For example J. B. Whitehead mention about exponential temperature rule in insulation, than astonishing remark from G.L. Moses saying the 10deg rule was well known in 40ties as rule for isolation. And You try to find out how it came into lifetime equation?

(https://books.google.pl/books/about/Impregnated_Paper_Insulation.html?id=G4xRAAAAMAAJ&redir_esc=y)

J. B. Whitehead and several German researchers knew, that insulation conductivity changes exponentially in temperature. It was after Johann Königsberger (father or semiconductor "Halbleiter") introduced 1906 a dissociation idea to study conduction in oxides and sulfides. (https://iopscience.iop.org/article/10.1088/0143-0807/10/4/002) One of the purpose for such assumptions was that Maxwell's relationship between conductivity and optical absorption was not valid here (PTB-Mitteilungen 135 (2025), Heft 1).

$$N = U \cdot e^{-\frac{Q}{RT}}.$$

Hierin bedeutet U die Gesamtzahl der Elektronen, N die Zahl der freien Elektronen, T ist die absolute Temperatur. (Königsberger, Annalen der Physik 1910: Vol 32 Iss 6)

One of the first experimental results with exponential fitting data was done by Foussereau (Compt. Rend. 95. 1882). Walther Nernst, Rasch & Hinrichsen, Vogel, Tammann, Fuchs, and many others knew about it in study electrolytic conduction of glasses.

Their equations share common a kind of Arrhenius factor 1/T.

Remark that Al2O3 the oxide layer of foil in electrolytic capacitors also conduct electrolytically (as was found by Carl Fritsch 1897; Annalen der Physik und Chemie 1897: Vol 60 Iss 2) and belongs to class II of glasses.

Most of this papers, books doesn't mention RGT rule, however it was Graham Lee Moses (Westinghouse Electric Corp.) whom presented 10deg rule in insulation.

$$R_2 = R_1 \times (0.44)^n \text{ for class A insulation.}$$
$$R_2 = R_1 \times (0.63)^n \text{ for class B insulation.}$$

where R_1 = resistance at temperature 1.
R_2 = resistance at higher temperature 2.
n = temperature difference (2 − 1) divided by 10.

(Moses G.L. Synthetic Insulation and the 10-Degree Rule *Westinghouse Engineering* 106 1945)

So there is a linkage not settled precisely between AH factor, conductivities and 10 deg rule. While the experimental results are needed to confirm some theoretical models.

Thermionic emission current

Thermionic emission study started after discovery that flame vapors conducts current. Arrhenius made such a experiments. Walther Nernst also knew theory of van't Hoff when assumed electrolytic conduction of hot flames before he patented a "Gluehlampe".

The extremely fruitful hypothesis of van't Hoff, that matter in the form of gas and in that of dilute solution has perfectly analogous properties, has so far been established in the most brilliant manner; and with the aid of the laws of gases a light has been

(Arrhenius: https://doi.org/10.1080/14786449108620108)

Harald Wilson (https://doi.org/10.1098/rspl.1903.0052) were using chemical reaction model in order to study conduction in thermal emission. Owen William Richardson used same idea after Wilson… That's a start of equations which are Arrhenius type… (linking reaction speed with some other physical behaviors like conductivity, leakage current).

Luckily Roger Stout (emeritus from On Semi/Motorola) mention 10deg rule in thermal runaway analysis of IC's.

We can now conclude that thermionic current follow Arrhenius relationship and could be linked with 10deg rule. Same applies for insulation resistance. Don't know yet how to link it… lets go further to discover it.

Conclusion

We see that reaction speed was a basis for some theories of conduction and thermionic emission current.

Montsinger, Akahira, Peek… HALT, ALT an old story…

Accelerated testing is an old discipline tracking back to August Wöhler and Olin Hanson Basquin endurance of train axels far long before it went to aeronautics and automotive industries. They found stress/strength relationship could be presented in exponential graph (American Society for Testing and Materials Proceedings, Vol. 10, 1910). This suggest something like in Arrhenius plot, yet we don't know it could be linked.

XIX century settled equation for fuses and started the circuit protection from fire. Robert P. Haviland use the example of time to burn out the fuse as an example of lifetime equation (Engineering Reliability and Long Life Design). Of course normal working cable is far from overcurrent yet one could wonder how long it will survive.

In electric technic the end of XIX century started the standardization, and also settled temperature insulation classes. It raises the question how much temp insulation should work, and how long will it survive?

The first of graph life-temperature was made by Steinmetz and Lamme in 1913. (10.1109/PAIEE.1913.6661165)

Historically we shall start with V. M. Montsinger (General Electric) 1930 discovery which bounded insulation age (tensile strength) with temperature. It was exponential law which doesn't looks like Arrhenius once at first glance.

$$Y = A\,e^{-mx}$$

Y = time.
A = constant.
e = base of naperian logarithm.
m = constant = (0.088).*
x = temperature deg. cent.

(10.1109/T-AIEE.1930.5055572)

Montsinger mention the same results were obtained by Vannevar Bush team which started the MIT insulation lab. His early results are dated 1921 while Bush 1923. The MIT lab was formed by famous Arthur von Hippel which escaped Nazi-Germany. This lab joined Hans Müller from ETH Zürich and made progress in Barium titanate theories. (P2011-1_JarrendahlKahr_JAW12_Hans_Mueller)

Better result in line with Arrhenius equation obtained Takeo Akahira and Tsunetaro Kujirai at thermogravimetric method (1925) using Kottaro Honda thermobalance.

$$\frac{dw}{dt} = Ae^{-E/RT}f(w)$$

(https://delibra.bg.polsl.pl/Content/18085/nr7_b2_1925.pdf)

(https://www.netsu.org/JSCTANetsuSokutei/pdfs/17/17-2-101.pdf)

Maybe Montsinger was not aware of their research, however Germans knew it. It was because inventor of thermobalance Kotaro Honda was Gustav Tammann (Göttingen) lecturer. As Tamman worked in metallurgy and was involved in many metallics used for example in resistors (Lagierungen). Tammann pioneered in research of oxidation by using optical methods to determine thickness (doi:10.1002/zaac.19201110107). It was bounded with chemical reaction speed, which later on bring Mr. Honda idea for preparing TGA equipment.

Worth to note is Takeo Akahira was a student college of Mr. Toyoda founder of Toyota, and influence much the engineering insulation industry in Japan.

(https://www.toyota-global.com/company/history_of_toyota/75years/text/taking_on_the_automotive_business/chapter1/section3/item1.html)

Here is important that Akahira insulation aging model is inline with Arrhenius kind relation.

The story is longer, one of the earliest mention in relation aging -reaction speed goes to Daikin 1948. (10.1109/T-AIEE.1948.5059649) However Daikin doesn't mention his equation are taken from Jacobus Vant Hoff XIX century book.

$$-\frac{\partial C}{\partial t} = k\,C^{n}$$

(1884 „Etudes de dynamique chimique". Hoff Jacobus Henricus) https://gallica.bnf.fr/ark:/12148/bpt6k9812616k

Luckily during WWII (1942) Walter Büssing published two papers relating Arrhenius law with aging of isolation. Büssing worked for Siemens Halske. (Archiv für Elektrotechnik Volume 36, Issue 12 and Issue 6, 1942)

Same year 1942 Heinrich Hess in his book mention Montsinger experimental law reminds reaction kinetics. Hess was working with Büssing and was a chief of electrical machines institute in TH Stuttgart.
https://www.vde.com/de/geschichte/karte/baden-wuerttemberg/th-stuttgart-elektrotechnisches-institut

Daikin and other researchers mention Swedish Goethe Malmlow whom idea was linking reaction speed with aging. Malmlow analyzed the pyrolysis of cellulose. We know that cellulose/paper as one of the early insulation for cabling and el-cap separators before newer material were developed. It looks that Malmlow moved from Sweden into Carnegie Institute technology US, while we don‘t know his roots, Royal Academy of Sweden or maybe Brown Boveri.

Does Daikin hear about Büssing ideas? We don’t know... but again Germany pioneered.

Weight loss and TGA a common point in el-caps life…

Todays some el-cap OEM especially Japanese are using a kind of thermogravimetric analysis in their testing of capacitor aging. The earliest mention found in Alwitt 1965 paper (Sprague 10.1109/TPMP.1965.1135396) describe the analysis of weight loss, while the pretty paper by Japanese in 1987 treats the weight loss in connection with capacitance drop. (https://doi.org/10.1541/ieejias.107.598) Of course the paper do not share equation or parametric drift models. As was shown the TGA story is very old and it explain why Japanese uses weight loss.

Weight loss was also treated by ANSYS reliability engineer (formerly DfR) Greg Caswell. (ASQ-Presentation-5-13-21 Evolution of Electrolytic Capacitors- Why a Reliability Engineer Should Know This)

SPRAGUE 510D EXTENDED TEMPERATURE CAPACITOR
ELECTROLYTE WEIGHT LOSS IN MILLIGRAMS/1000 HOURS

Temperature	Mg/1000 Hrs 6 × 11 MM	(Referenced to 125°C) Multiplier	Calculated Life (Hours)
125°C	12.0	× 1	1000
105°C	6.0	× 2	2000
85°C	1.5	× 8	8000
55°C	0.50	× 24	24000
25/30°C	0.25	× 48	48000

(old Sprague paper showing weight loss vs lifetime)

Power Law

Voltage is second parameter which affects the aging, after temperature.

$$L_1 = L_2(E_2/E_1)^n$$

(10.1109/T-AIEE.1944.5058861)

In technics, especially cable, dielectric strength and breakdown is one of important factors dimensioning the cable. Today hardly one remember it started with cellulose paper as insulation sheath at cabling. Power law says that strength of breakdown time behaves like some nth power of applied voltage.

J.B. Whitehead and some of his colleagues says Power Law was nothing new (1940) once wondered whom introduced the Power law. We tracked that F.W. Peek showed power rule about 1915 and guess his development was earlier.

$$g = g_s\left(1 + \frac{a}{\sqrt[4]{T}}\right)$$

(Dielectric Phenomena In High-voltage Engineering Ist ed 1915)

The power law is used up today in MLCC, capacitors and isolation…. Sounds interesting, don’t You think?

Mention about power law has great importance, its older than other research in aging, and its use in today lifetime equations.

Power law explains how breakdown strength follow with time.

The idea of Peek was brilliant once, instead of using linear scale he transformed time axle into reciprocal of nth degree square of time and obtained linear breakage vs time (transformed) result.

Worth here to mention Montsinger tested voltage time effect of breakdown and with Vladmir Karapetoof he settled some information about pyroelectric effect with theory of Karl Willy Wagner (Thermal breakdown). The idea of thermal breakdown started in 1912 when Walker Miles (Journal. I. E. E. 1912 vol 49) put the idea about thermal runaway. He was the first which noted that temperature control is missing in previous insulation tests.

Today's partial discharges are treated responsible for a long time aging. (AIEE Journal 1924 Vol 43 Iss 12)

The thermal runaway in concept presented by Roger Stout, doesnt differ a much from the Miles concept.

It must be added that Wagner results 1922 (10.1109/T-AIEE.1922.5060783) were using 3 kind of resistivity vs temperature model. With one of them was Berthelot style, which is in line with Stout leakage current model.

A power law with statistics of breakdown were show in M. C. Holmes. (10.1109/T-AIEE.1931.5055975)

Montsinger didn't mention in his thermal aging paper about part statistics, however he should be aware of it, since Bush and Moon analyzed the puncture voltage with gaussian normal distribution. (10.1109/T-AIEE.1927.5061443)

Conclusion

In 40ties industry settled the relation between AH reaction temp-coefficient and aging law.

In capacitor industry t started with paper capacitors similar to cable insulation which was also paper once at start.

Why the lifetime equation should be named Berthelot

Such an broad picture helped us imagine what was going on.

In chemistry it was complicated much more, starting from Berthelot, van't Hoff, Arrhenius, than Max Bodenstein, Max Trautz and the others. There were several equations for temp-coefficient of reaction speed.

Luckily Keth Laidler (https://doi.org/10.1021/ed061p494) summarized it pretty much and explained which are more general like Harcourt Essom one, and why the different equations results fitting with small error in limited temperature range.

Why….?

Because the engineering equation popular used in technics is not Arrhenius is much of Berthelot style.

Keith Laidler and H.S. Blanks (https://doi.org/10.1016/0026-2714(80)90211-5) pointed out this discrepancy, and explained why its valid. It comes that in low interval of temperature – like in capacitor, let say 30C till 130C max the Arrhenius equation gets very similar results with Berthelot.

Interesting note about it was found in Propellant analysis (AD0763879), when guys conclude that they wanted use Arrhenius equation when by accident a French team visited them and told them, „why don't You use Berthelot???"

The kind of approximation was shown by Sam Parler (https://www.cde.com/resources/technical-papers/multipliers.pdf), Hiroshi Shiomi (10.1109/IRPS.1965.362316) and in Joachim Boecker doctoral thesis (https://ei.uni-paderborn.de/fileadmin-eim/elektrotechnik/fg/lea/Lehre/EA/Dokumente/Skript_Elektrische_Antriebstechnik.pdf).

In 1937 G. Richter worked on "magnetische nachwirkung". Here he used temp dependence similar to Arrhenius and concluded " Die verschiebung setzen wir proportional T oder -1/T, was wegen der Kleinheit des Intervalls praktisch dasselbe bedeutet".(doi:10.1002/andp.19374210705)

He says in short T intervals the shift of ch-tic is proportional to T or -1/T

Very similar sentence was also found at Andreas Gemant/Gyemant (1930) book "Elektrophysik der Isolierstoffe". We can now understand why in lifetime equation Berthelot T or Arrhenius 1/T is valid. Again, as world is small... Gemant (Siemens Schuckerwerte) worked also with Takeo Akahira.

$\sigma = \sigma_0 e^{-a/T}$ and for small temperature differences $\sigma = \sigma_0 e^{bt}$ (see A. Gemant „Elektrophysik der Isolierstoffe")

We shall point out that this approximation should be judged. It was done by medicine paper (Jakob Jantig, 10.1016/s0969-806x(99)00403-x) and Lambert when they said the Berthelot rule can bring some kind of safety in equation… He names it conservative and such a model will give less hours than in reality would be consumed.

> dation. The line labeled 'model' corresponds to a model such as the $Q_{10}=2$ model. The important point to be clarified from this generic plot is the relationship between the slopes of the two lines. The slope of the reality line is steeper than the model, i.e., at a given temperature, the real AF is greater than the AF predicted by the model. This means that the material remains functional longer than the model predicts. This is a conservative situation.

(Lambert, https://doi.org/10.1016/S0969-806X(99)00403-X)

The idea behind Berthelot is to transform Arrhenius equation into an approximate Berthelot one, then use a log base transform getting the RGT 10deg rule. The Taylor expansion is proposed by Author.

$$k = A\mathrm{e}^{-\frac{E_A}{k_b T}} = A\mathrm{e}^{-\frac{E_A}{k_b(T_0+\Delta T)}} = A\mathrm{e}^{-\frac{E_A}{k_b T_0(1+\Delta T/T_0)}} \approx A\mathrm{e}^{-\frac{E_A(1-\Delta T/T_0)}{k_b T_0}}$$
$$= A\mathrm{e}^{-\frac{E_A}{k_b T_0}}\mathrm{e}^{+\frac{E_A \Delta T}{k_b T_0^2}} = A'\mathrm{e}^{c\Delta T}$$

Prof. Dr.-Ing. Joachim Böcker

Taylor's Series of $\frac{1}{1+x}$

$$\frac{1}{1+x} = 1 - x$$

Conclusion without proof

Capacitor OEMs can use lifetime equation with 2 base an 10 deg rule assuming it lays on the safe site of approximation. In other words the real life behavior of unit in a field is better than obtained from the equation. It would be than inline with a safety factor introduced by Robert Lusser in 1958.

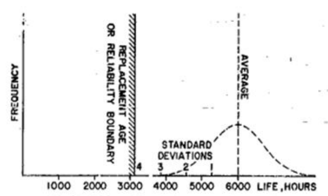

Fig. 11. Safety Margin Between Mean Life and Replacement Age Robert Lusser (Robert Lusser: https://apps.dtic.mil/sti/tr/pdf/AD0212476.pdf)

But... but… why the relation with reaction speed?

One of the best answer were made by Italian Professor Luciano Simoni (10.1109/TEI.1973.299248) and American TGA expert Derek Toop (10.1109/TEI.1971.299128). They considered in general function of property in relation to determine end of life.

Basic idea of life consumption versus reaction speed was pretty well presented by Luciano Simoni, when the property is diminishing it calls for similarity with reaction speed and reaction when one of contributor is changing to another… The fail (reaching lifetime) means when some critical amount of the chemical has reacted.

$$\text{critical amount} = \text{rate} \times \text{time to failure}$$
$$\text{time to failure} = \frac{\text{critical amount}}{\text{rate}},$$

(Horst Rinne https://doi.org/10.1201/9781420087444)

The property which change with constant rate (like a weight loss in example) allows calculation reaching critical/threshold amount aka "life-time".

This mean aging rate and life are inversely proportional! It's all the mystery of Arrhenius.

Most of the models assume rate of variation of property is kept constant under constant stress.

Cumulative damage a brief story

In most conditions the load and temperature is changing during item life. So the question is how calculate cumulative damage. Americans populated the Miner rule (https://doi.org/10.1115/1.4009458), sometime called Palmgren-Miner. Worth note is that Büssing used cumulative rule similar to Arvid Palmgren before Miner did it. That's a long path here, cause cumulative damage is not good described in capacitor application specs. Büssing showed that lifetime equation shall be integrated in a case of changing temperature. Later on this was analyzed by many of researches. One of them was Joseph Ben Uri which was minister of Israel in later years. (https://doi.org/10.1049/pi-c.1960.0022)

We don't know the Miner rule is valid in el-caps. User should be care that under varying condition one cannot simple add percentage of consumed life. Normally under the time changing temperature the lifetime equation should be integrated. Maybe it could be averaged for such a purpose to avoid integration and make only easier summing up.

NASA studied and translated Palmgren paper (NASA TT F- 13460) about cumulative damage, they translated his German paper, and it was found the idea of Palmgren is very similar to that of Büssing and Miner in cumulative damage (a linear case).

$$\sum \frac{\Delta t_i}{\tau_i} \leq 1. \qquad \int_0^{\tau} \frac{dt}{\tau} = 1. \qquad \tau = F(f(t))$$

(Büssing idea, for discrete temperatures, and when the temperature varies continuously with time the integral is used)

have been consumed. If the bearing is exposed to a certain load for a run of m_1 million revolutions where it has a life of n_1 million revolutions, and to a different load for a run of m_2 million revolutions where it will reach a life of n_2 million revolutions, and so on, we will obtain

$$\frac{m_1}{n_1} + \frac{m_2}{n_2} + \frac{m_3}{n_3} + \cdots = 1 \qquad (11)$$

(Palmgren paper translated by NASA)

$$L = \frac{\tau}{\int_0^{\tau} \frac{dt}{L[stressors(t)]}}$$

(CDE follows Bussing idea)

(https://www.cde.com/resources/technical-papers/TransientModelingOfLargeScrew-TerminalAluminumElectrolyticCapacitors.pdf)

The Palmgren idea was a reference to search by the author such a mirror relation in electrical industries. One can see the idea of Palmgren were reused by Büssing.

Comparing idea of Büssing vs Alfred M. Freudenthal cumulative damage. Here also some analogies could be kept. The Palmgren rule is used up today in life time cumulative damage. **Is it valid for el-caps too... ?**

$$D_{VS} = \int_{N_{OS}}^{V_S} (dD/dN)\, dN = \int_{N_{OS}}^{V_S} f(N)_S\, dN = 1.0$$

(doi:10.1016/s0065-2156(08)70372-7)

Here: D=1 means reaching fracture == End of Life, so those concepts are familiar.

What about life statistics?

The problem of aging was attacked with searching the origins of equation in capacitor technology. Unfortunately there is no one common standard nor norm (for capacitor) how to calculate the acceleration factor and treat it with statistics.

All of the ALT/HALT tests and endurance are based on limited samples, thus its obvious such an approach call for statistics or distribution analysis.

It was found that in contrary to capacitor industry the isolation has shared in 1956 a least square method to fit life equation with Arrhenius (10.1109/AIEEPAS.1956.4499319). Later on it was superseded with log-normal (IEEE-101). Nelson commented log-normal model is pretty well suitable for isolation in „natural way“. However he doesn't explain why such an choice was done… (10.1002/9780470316795) Maybe a bit of light can be found in Goldenberg paper 1961 discussion (10.1109/AIEEPAS.1961.4501180) when he says that when life is normally distributed, than log life is also normal distributed (with some limitations).

Of course OEM specs and life equation do not tell You easily which distribution they used.

Starting from 50ties thanks to Epstein the exponential was popularized, than Bell-labs started log-normal usage. Mean time Weibull grew in popularity cause its universality. Exponential approach is still used today like in JIS C5003, however at 60ties NIST (Electronic Industries 1960-12: Vol 19 Iss 12) pointed out lack of robustness from exponential distribution test when the failure rate is growing like in wear out phase. Thus in order to avoid errors industry moved to log normal and Weibull.

Worth note here, Cornell Dubilier uses EIA IS-749 which other companies do not use. This standard says that 10% of capacitors in life test must fail (reach the defined EOL threshold). We searched and found that 10% was first popularized by Palmgren in ball-bearing industry to avoid costs at long term. It was a compromise between test time and costs.

maximum, or for an intermediate service life between these two extremes....In order to obtain a good, cost effective result, it is necessary to accept that a certain small number of bearings will have a shorter service life than the calculated lifetime, and therefore the constants must be calculated so that 90% of all the bearings have a service life longer than that stated in the formula. The calculation procedure must be considered entirely satisfactory from both an engineering and a business point of view, if we are to keep in mind that the mean service life is much longer than the calculated service life and that those bearings that have a shorter life actually only require repairs by replacement of the part which is damaged first." (https://ntrs.nasa.gov/api/citations/19970025228)

Palmgren like in EIA IS-749 used 10% failure L10 which could suggest B10 from Weibull distribution (after Abernethy B10 "Bruchaneinleitzeit" – failed once). Its very important say that Palmgren worked with Walloddi Weibull to treat statistically the ball bearings life, both worked and supported Swedish SKF. (http://km.fgg.uni-lj.si/PREDMETI/sei/Ljudje/weibull.htm) (https://sv.wikipedia.org/wiki/Waloddi_Weibull)

Of course EIA IS-749 doesn't suggest distribution, but the 10% value is in line with ball bearing L10. Sounds interesting...

What is missing in LT equation is the relation to reliability level like in ball bearings, in bearing industry one can get the L10 life and with some factor recalculate life at different reliability level. **In capacitor specs there is however no such an information defined (!!!)**

Reliability %	$L_{n\text{m}}$	a_1
90	$L_{10\text{m}}$	1
95	$L_{5\text{m}}$	0,64
96	$L_{4\text{m}}$	0,55

(ISO-281)

As the B10/L10 suggest Weibull, one should say that first Weibull 1939 paper has mistakes and grammar errors, as says Horst Rinne, so it was not popularized so early. However Rinne showed that Weibull was nothing new and originated from R. A. Fisher EVA and from sand-corn distribution. It was Rosin & Rammler work on grain size, which developed a kind of Weibull statistics. Interesting thing as they used Pearson (1893) equation for extremes IX century results… 😊

(https://doi.org/10.1201/9781420087444)

On the other side the log normal was used extensively by Bell Labs. Log normal started with von Kapteyn discovery, but it was criticized by Karl Pearson on early stages and his great authority slower the implementation.

In MLCC NASA specialists are using a kind of grain size distribution which reminds somehow that of Roisin Rammler. Japanese have found correlation between grain size and dielectric breakdown strength. On the other hand many grain size studies start from Russia with example of Kolmogoroff 1941 paper showing mathematical background. The log normal together with Weibull are a kind of basis, for grinding, milling and grain size distribution. (https://nepp.nasa.gov/files/24398/Liu_2013_G11_Presentation.pdf)

This judges log normal, and same for Weibull. In one of the old book („Reliability training text") we can found log normal Weibull and gamma are very similar an thus log-normal were taken into reliability studies.

The lifetime equation looks like empirical law and normally user do not know the statistics behind.

The question is …why? is it secrecy?

Using Huai Wang (Aalborg Denmark) expertise he concludes that when no reliability level, confidence level is defined, one cannot simply compare the reliability. So the end user cannot compare the one capacitor lifetime with the other…!!

(https://www.psma.com/sites/default/files/uploads/files/Failure%20Modes%20(Theoretical%20Input)%20(Huai%20Wang%2C%20Aalborg%20University).pdf)

Good example here are endurance tests… Taking a look at endurance test result no one of items has reached EOL threshold. The question is… how shall One believe what life equation says? When no of the item has failed? It's something missing here…

Some scientists from 80ties commented Weibull and log normal fit the test data equally well, however the behave quite different. It was Evans Ralph saying that log normal fit rate is falling down so it cannot be named wear out distribution.

mean life. For very small values of skewness (σ<0.2) most of the population is dead before the Ponce de Leon point is reached. Nevertheless, if one waits long enough, the logNormal distribution always has a decreasing hazard rate and thus can not ever be termed a wearout distribution. (Ralph Evans) (Ponce de Leon)

(https://es.wikipedia.org/wiki/Juan_Ponce_de_Le%C3%B3n)

(https://ethw.org/w/images/8/82/Vol_15_-_Issue_1_-_Jan_1970_-_IEEE_Reliablity_Group_Newsletter.pdf)

One can try handle the topic with support of some paper, good choice is Franck Bayle (10.1002/9781119610717) from Thales. He has concluded that „**reliability tests are done with no failure an rarely judged why"**. He comments about purpose of the chi-squared estimator. The same estimator is used in JIS C5003 used by Japanese el-caps OEMs. It's a kind of Epstein rule ("Estimation from life test data" and "Life testing" paper). The issue is, this estimation was taken from test with not failure, assign exponential distribution has no sense than and thus a chi square is use to „guess" estimate the limit and lower confidence level.

$$\hat{\lambda}_C = \frac{x^2_{(C;2)}}{2t}$$ (ADA026353) $$\lambda u = \frac{0.917}{T}$$ (Capacitor OEM upper failure rate)

Exponential Zero Failure Estimates

Percentile	Estimate
50	0.6931/nT
60	0.9163/nT
80	1.6094/nT
90	2.3026/nT
95	2.9957/nT
97.5	3.6889/nT
99	4.6052/nT

In old paper form Sprague (Technical paper No. 64-2) it was shown a modified life equation with R% failure. We handled and tracked a story in order to understand why... An Younger electrical engineer Lindquist from US in his master thesis pointed out first MIL 217 (1962) which shows Arrhenius type (or better Berthelot) graph connecting failure ratio with temperature. On the other hand Sprague FIT graph shows 60% confidence level which is in line with JIS-C5003 for exponential case, but… we know for wear-out phase the log-normal or Weibull should be used… again a kind of mystery exists. The zero failure case with an estimator was handled by Zhaofeng Huang (Rockwell Rocket) for Weibull data and TRW guys (Welker, Lipow) for exponential case. The life equation suggest wear out mode, while failure rate is taken from hazard mode and there is no connection between them...

(Lindquist: https://ir.library.oregonstate.edu/concern/graduate_thesis_or_dissertations/f4752m120)

So the old Sprague approach looks to be still valid. However 60% confidence is like fifty-fifty coin tap result will be higher 50% and lower 50% than the estimator says. What worth the lifetime definition will have in such a case?

$$\frac{L_1}{L_2} = \frac{R_1}{R_2} x \left(\frac{V_2}{V_1}\right)^n x \ 2^{\left(\frac{T_2 - T_1}{k}\right)}$$

L = time to a given % failures
R = % failures on which L is based
V = d.c. voltage
T = temperature (°C)
n = power law exponent
k = temperature rule constant

(Sprague Technical paper No. 64-2)

Sprague approach links somehow the failure rate with life which todays equation miss…

60ty percent confidence level from Sprague suggest exponential statistics…. It's in line with Japanese OEM data showing FIT/ratio according to JIS C5003… with the mentioned Epstein estimator. Yet the book from Franck Bayle (Thales) says without failure we do not know the statistics behind… and estimator calculates the upper bound of failure rate. Of course the information that part didn't failed is valuable.

The only confidence level was presented in CDE paper and using gaussian normal distribution.

> 1.2L with 90% confidence. Stated another way, our goal is to say with 90% confidence that no more than 10% of the capacitors will have failed from wearout before t=L. Due to the expected distribution, this would be about the same as saying with 90% confidence that no more than 1% will have failed from wearout before t=0.9L.

(Cornell Dubilier, https://www.cde.com/resources/technical-papers/reliability.pdf)

The judgment for choosing normal distribution was not done in CDE paper, yet it was found (see Lindquist) that normal distribution could be used as wear out distribution. Lindquist math basis was taken from Erich Pieruschka and his colleague Robert Lusser books. Those two reliability engineers were famous from working in V1 project, and supported JPL in famous Werner von Braun team. (https://en.wikipedia.org/wiki/Robert_Lusser)

Nevertheless, the life equation from OEM has never information about confidence level and reliability level…

The only mention about non zero failure was found in ANSYS papers:

(https://www.ansys.com/content/dam/product/electronics/sherlock/thermal-management-solutions-how-hot-is-too-hot.pdf)

> Unlike most other component technologies, the lifetimes defined by electrolytic capacitors are more conservative than MTTF, with actual failure rates at lifetime between 1 to 5%. Designers have typically applied a 50% derating to the ripple current, which greatly extends the lifetime.

(ANSYS)

So as far we know, that we still don't know. And we found that statistics is rarely done when no failure exist. On the other hand, there are many papers, how to relate Arrhenius with log-normal, Weibull etc. This is called by H. Rinne a model with covariates, and belong to proportional hazard model developed by Cox in 1972. Worth note is that (by Elsayed) that proportional model is distribution free however can be bounded with other equations (statistical model with stresses…).

The analysis of life data make step in progress when in about 80ties researchers switch form catastrophic into degradation failure. They introduced a kind of parametric model to establish a drift function vs time. Interest is that today statistics/reliability tools have possibility to link AH (Arrhenius) model with either log Normal, Weibull, and parametric models. Since most degradation data from endurance show threshold of ESR/capacitance/tang delta it suggests that multi-variational analysis tools should be used.

Open Questions for OEMs

There is a missing point relating wear out, hazardous mode with LT equation. Now its time to open some questions, it's not easy prepare them in clear way. The misunderstood can still exists.

As was mention the Automotive engineer is obligated to calculate lifetime of capacitor in application and review it with end customer (Automotive OEM). It could be linked with electrolytic capacitors in DC link for inverter as an example. Yet the lifetime equation as shared with capacitor OEM has itself many misunderstoods:

- Usually, OEM shares the lifetime equation with a kind of power law, Arrhenius mix (or other):

  $$L = L_0 \cdot K_T \cdot K_R \cdot K_V$$

L	Total Lifetime
L_0	Lifetime under Rated Ripple Current at Upper Category Temperature (see catalogue)
K_T	Temperature Factor
K_R	Ripple Current Factor
K_V	Voltage Factor

- However, there is no information how to understand its results (from equation) because:
  - The life is calculated from limited number of samples from population and measured in limited time (with failure or not!! → zero failures in most specs)
    - This involves the statistics.
    - The statistics of lifetime equation is never defined (shared with the END engineer in spec.).
    - Most OEM specs shows only endurance (applied ripple, high temp, and nominal voltage) 1000h with no failure of items!! Is there a link „Endurance" == „Lo" base life time?
    - User do not know the relation between LT equation and failure modes (wear out, hazard)
- The END users can calculate the life from equation, but he doesn't know:
  - *The confidence limit of this calculation.*
  - *The reliability level. (how many items will fail 1%/1ppm/zero or other)*
  - *Do not know if the results is the mean life, or a kind of minimum life (worst case).*
  - *When the End user looks at endurance data, he seems no item has failed, so how can he trust the life equation?*
  - *End user can only guess that lifetime equation has some safety factor, but he don't know about it….*
- Most OEMs do not define nothing about the cumulative damage rules:
  - Is it linear like in Palmgren Miner rule (see the equation) ...???
  - Or nonlinear...???
  - $$\sum^{n} (t_i/L_i) = 1.$$
- The LED industry and ball bearing industry usually links the life with „Bxx" reliability level. While capacitor OEMs do not… (in clear way).

SUMMARY

The paper presented the story of lifetime equation and his origins with chemical reaction rate. This empirical equation was linked than with life statistics. Since today LT equation shared by OEM‘s doesn’t mention about part statistics the open question was addressed.

As Greg Caswell Ansys/DfR comments: failure definition between OEMs can vary, lifetime can be with or without ripple current, probability of failure after lifetime is never defined (because test to 0 failures is only shown) and degradation of seal/rubber bung is not addressed. This judges the questions presented.

END